A Novel Traffic-adaptive Spectrum Leasing Scheme between Primary and Secondary Networks

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Abstract—**Spectrum leasing has been widely regarded as one of the most effective ways to improve the utilization of limited spectrum resources. The existing literatures normally regulate secondary users (SUs) to lease unused licensed spectrum from primary users (PUs) for indefinite or predetermined time length, which does not adapt to real-time traffic demands of SUs and may limit the utilization of licensed spectrum channels. In view of this, the present paper proposes a novel traffic-adaptive spectrum leasing scheme, which allows PUs and SUs to negotiate leasing periods with variable time length such that SUs can continuously utilize the leasing channels for transmitting dynamically generated secondary packets until their transmission buffers become empty. Following this scheme, we further formulate the utilities of both PUs and SUs in traffic-adaptive spectrum leasing based on M/M/*k* queues and derive the unique sub-game perfect equilibrium of this scheme based on the Stackelberg game model. Numerical simulation shows that, compared with the existing spectrum leasing schemes based on cooperative relaying, the proposed scheme can concurrently afford both PN and SN with higher leasing utilities, encourage them to join spectrum leasing, and achieve a better utilization of limited spectrum resources.**

I. Introduction

Spectrum leasing has been widely regarded as one of the most effective ways to improve the utilization of limited spectrum resources [1]. It allows primary users (PUs) to lease unused spectrum to secondary users (SUs) for receiving finacial payoff. As a return, SUs can legally utilize the leased bandwidth for data transmission. In cognitive radio, spectrum leasing can also save the time and energy of SUs for sensing the holes of licensed spectrum. In practice, various Mobile Virtual Network Operators (MVNOs), i.e., GSM Nation, Karma, and Voyager, have provided mobile services based on the spectrum leased from Mobile Network Operators (MNOs).

Existing literatures have proposed various spectrum leasing schemes for specific application scenarios. For example, the spectrum leasing schemes ([2], [3]) in the *underly* fashion only allow SUs to lease the licensed spectrum given that their interference to PUs cannot exceed a prescribed threshold. Since the threshold may limit the transmission performance of SUs, more research works have focused on the *overlay* spectrum leasing for maximizing the benefits of various leasing parties, e.g., PUs, SUs, and spectrum brokers, and, meanwhile, maintain a reasonable fairness among them. In the spectrum leasing schemes ([4], [5]), multiple SUs under centralized control provide cooperative relay service for primary transmission and, in turn, obtain a temporary time period for legally utilizing the licensed spectrum for secondary transmission. The existing schemes ([6], [7]) further allow SUs to compete for the opportunity to serve as cooperative relays under appropriate distributed controls. The main advantage of these schemes is that they enable PUs and SUs to achieve a win-win solution, i.e., the performance of primary transmission benefits from the cooperative relaying service offered by SUs, which can temporarily obtain licensed spectrum for secondary transmission.

On the other hand, these schemes also share a common shortcoming, i.e., the temporary spectrum resources obtained by SUs cannot adapt to the dynamically changing secondary traffic demand very well. For instance, in the spectrum leasing schemes ([6], [7]) under distributed control, it is difficult for those SUs, which have worse channel conditions than other SUs, to obtain the relaying opportunity for primary transmission and hence cannot guarantee the QoS for their data traffics. More importantly, even if each SU can successfully lease the leasing channel for secondary transmission, the length of its leasing period is normally predetermined before the real secondary transmission begins and cannot be changed thereafter. When the SU generate data traffics with non-uniform length in a fully random fashion, a fixed-length leasing period will definitely mismatch with the real-time secondary traffic demand. To offer a better QoS for secondary transmission and improve the utilization of limited spectrum resources, a natural intuition is to perform spectrum leasing in a more dynamic way, i.e., SUs can lease the licensed spectrum for variable-length time periods based on their real-time traffic demands.

In view of this, the present paper proposes a novel traffic-adaptive spectrum leasing (TASL) scheme, which allows PUs and SUs to negotiate leasing periods with variable time length such that SUs can continuously utilize the leasing channels for transmitting dynamically generated secondary packets (SPs) from the time when SUs accumulate a certain number of buffering SPs to the time when the transmission buffers of all SUs first become empty. Following this scheme, we further formulate the utilities of both PUs and SUs in traffic-adaptive spectrum leasing based on M/M/*k* queues by considering both the revenues of packet transmission and the costs for channel handoff and packet buffering. Finally, to avoid possible multiple equilibriums, we apply two rules, which can enhance the QoS of primary and secondary transmission, to derive the unique sub-game perfect equilibrium of the TASL scheme based on the Stackelberg game model. Numerical simulation shows that, compared with the partially traffic-adaptive spectrum leasing (PTASL) scheme, which generalizes the existing spectrum leasing schemes ([4], [5]) based on cooperative relaying, the proposed scheme can concurrently afford both PN and SN with higher leasing utilities, make them more willing to join spectrum leasing, and achieve a better utilization of limited spectrum resources.

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The remaining of the present paper is organized as follows. Section II describes the system model for the proposed TASL scheme and Section III formulates the utilities of PN and SN in this scheme based on M/M/k queue. Based on these utilities, Section IV further derives the unique sub-game perfect equilibrium of this scheme based on the Stackelberg game model [8]. Numeric simulation in Section V then compares the performance of the TASL and PTASL schemes and Section VI finally concludes the main contribution of this paper.

II. System model for traffic-adaptive spectrum leasing

This paper considers a wireless communication system, which consists of one PN and one SN. In this system, the SN, composed of one secondary access point (SAP) and multiple SUs, is a single-hop ad hoc network lack of legal spectrum resources, while the PN, composed of one primary base station (PBS) and multiple PUs, has totally N, $N \geq 1$, disjoint licensed spectrum channels with uniform bandwidth for primary transmission. To satisfy its communication requirement, the SN has to pay a price to lease a certain number of licensed channels from the PN for secondary communications. Meanwhile, to maximize its benefit without affecting the basic requirement of primary transmission, the PN can lease part of the N licensed channels to the SN for monetary revenue.

Once the SN acquires a certain number of licensed channels from the PN, the SAP will regulate all SUs to sequentially transmit their buffered packets over the acquired channels according to an appropriate order, which may include various traffic parameters into consideration, such as traffic type (e.g., real-time or non-real-time), traffic emergency, and user priority. Whenever a SU generates a data packet, it has to first buffer this packet and then report it to the SAP via a dedicated narrow-band control channel, which can be regarded as being free of interference. Only after the SU obtains an approval from the SAP via the same channel, it will begin to transmit the buffered packet over the channel specified by the SAP. Thus there exists no transmission collision within the SN and PN. For the simplicity of further derivation, we also assume that the PBS coordinate the packet transmission among PUs in the same way as the SAP does.

Because the PN and SN have different interests, the key problem is how to form a mutually beneficial spectrum leasing agreement between them. For this purpose, the present paper proposes a dynamic spectrum leasing scheme for the SN to lease a certain number of licensed channels from the PN according to the real-time traffic demands of all SUs. In this scheme, if the SN successfully leases N_S, $1 \leq N_S < N$, licensed channels, the infinite time of these channels will be synchronously divided into two types of time periods, namely *leasing* and *buffering periods*, which alternatively appear in the time axis as shown in Figure 1. A leasing period begins when the SN has buffered exactly n, $n \geq 1$, secondary packets (SPs) and ends when the transmission buffers of all SUs become empty, while the gap between any two adjacent leasing periods is a buffering period for the SN to accumulate buffering SPs. Meanwhile, the PN can transmit primary packets (PPs) over the N_S leasing channels in each buffering period and utilize the remaining $N_P = N - N_S$ reserved channels for primary communication at any time. Because of the random generation of SPs, any two buffering periods may have different time lengths and so do any two leasing periods.

Assume that the PN (*resp.* SN) generates PPs (*resp.* SPs) according to a Poisson process with the rate λ_P (*resp.* λ_S) and the transmission time of each PP (*resp.* SP) over a licensed channel follows an exponential distribution with the mean of μ_P (*resp.* μ_S). Thus the average time interval between the generation of any two consecutive PPs (*resp.* SPs) is $1/\lambda_P$ (*resp.* $1/\lambda_S$) and the expected length of each buffering period in Figure 1 is n/λ_S. Following this assumption, the transmission of SPs over the N_S leasing channels can be regarded as an M/M/N_S queue, where all SPs forms a queue and the N_S leasing channels serve as the N_S parallel servers for this queue, while the transmission of PPs over the N (*resp.* N_P) licensed channels during each buffering (*resp.* leasing) period can be regarded as an M/M/N (*resp.* M/M/N_P) queue, where all PPs forms a queue and the N (*resp.* N_P) licensed channels serve as the N (*resp.* N_P) parallel servers for this queue.

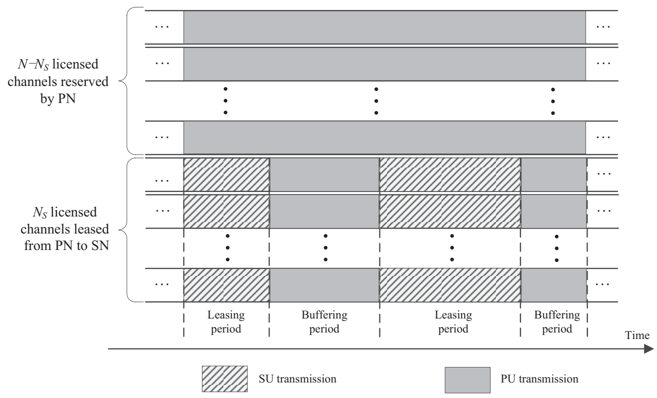

Fig. 1. In the proposed traffic-adaptive spectrum leasing (TASL) scheme, the infinite time of the N_S licensed channels leased from the PN to the SN are synchronously divided into leasing and buffering periods with variable time lengths, which can fully adapt to the real-time secondary traffic.

From [9], the long-run probability π_k, $k \geq 0$, for the PN to buffer exactly k packets at the beginning of a buffering or leasing period can be expressed as:

$$\pi_k(C,\rho) = \begin{cases} \dfrac{\frac{(C\rho)^k}{k!}}{\sum_{k=0}^{C-1}\frac{(C\rho)^k}{k!} + \frac{(C\rho)^C}{C!(1-\rho)}}, & if\ k \leq C \\ \dfrac{\rho^i C^C}{C!}\pi_0(C,\rho), & if\ k > C \end{cases} \quad (1)$$

where $C = N_P$ and $\rho = \frac{\lambda_P \mu_P}{N_P}$ for the beginning of a buffering period, while $C = N$ and $\rho = \frac{\lambda_P \mu_P}{N}$ for the beginning of a leasing period.

Without loss of the generality, assume that both PN and SN are honest in the execution of the proposed spectrum leasing scheme. That is, both PN and SN would like to exchange their utility parameters with each other and the SN does not cheat on the beginning and ending time of each leasing period. Following this assumption, Section III will first formulate the utilities of both PN and SN in this scheme and Section IV will then derive the equilibrium solution for the scheme.

III. Utilities in traffic-adaptive spectrum leasing

Denote by $E[T_L]$ and $E[T_B]$ the average time lengths of all leasing and buffering periods, respectively, and let $E[T_{pd}] = E[T_L] + E[T_B]$. In order to meet the QoS requirement of secondary transmissions, the SN should achieve an average transmission rate no smaller than a prescribed lower bound $\overline{R}_{S,min}$, i.e., $N_S\overline{R}_{S,c}\frac{E[T_L]}{E[T_{pd}]} \geq \overline{R}_{S,min}$, where $\overline{R}_{S,c}$ is the average transmission rate of a SU over a licensed channel. Meanwhile, to avoid the overflow of SU buffers in a long-run period, the SN should have the average time interval for generating consecutive secondary packets (SPs) no smaller than the average transmission time of each SP, i.e., $\frac{1}{\lambda_S} \geq \frac{\mu_S E[T_{pd}]}{N_S E[T_L]}$. Thus we have

$$N_S \geq max\Big\{\frac{\overline{R}_{S,min}E[T_{pd}]}{\overline{R}_{S,c}E[T_L]}, \frac{\lambda_S\mu_S E[T_{pd}]}{E[T_L]}\Big\}. \quad (2)$$

Similarly, to guarantee the QoS requirement of primary transmissions, the PN should achieve an average transmission rate no smaller than a prescribed lower bound $\overline{R}_{P,min}$, i.e., $\overline{R}_{P,c}\frac{N_P E[T_L]+NE[T_B]}{E[T_{pd}]} \geq \overline{R}_{P,min}$, where $\overline{R}_{P,c}$ is the average transmission rate of a PU over a licensed channel. Meanwhile, to avoid the overflow of PU buffers in a long-run period, the PN should have the average time interval for generating consecutive primary packets (PPs) no smaller than the average transmission time of each PP, i.e., $\frac{1}{\lambda_P} \geq \frac{\mu_P E[T_{pd}]}{N_P E[T_L]+NE[T_B]}$. Because $N_P = N - N_S$, we have

$$N_S \leq min\Big\{\frac{(N\overline{R}_{P,c} - \overline{R}_{P,min})E[T_{pd}]}{\overline{R}_{P,c}E[T_L]}, \frac{(N - \lambda_P\mu_P)E[T_{pd}]}{E[T_L]}\Big\}. \quad (3)$$

Thus the targets of both PN and SN in the traffic-adaptive spectrum leasing are to maximize their transmission benefits subject to the constraints (2) and (3).

Because of the random generation of SPs, the expected utility of the SN depends on both $E[T_L]$ and $E[T_B]$ and so does that of the PN. Since the average time interval for two consecutive generations of SPs is $\frac{1}{\lambda_S}$, we have $E[T_B] = \frac{n}{\lambda_S}$. Meanwhile, to derive $E[T_L]$, denote by W_j, $j \geq 1$, the time period starting from the first time when there are totally j SPs buffered or in transmission and ending at the first time when no SP is buffered or in transmission. Thus $E[T_L] = E[W_n]$. From [10] and [11], we have

$$E[T_L] = E[T_n] = \sum_{j=1}^{\infty}\rho_j + \sum_{m=1}^{n-1}[(\prod_{k=1}^{m}\frac{\mu_k}{\lambda_S})\sum_{j=m+1}^{\infty}\rho_j], \quad (4)$$

where $\rho_j = \dfrac{\lambda_S^{j-1}}{\mu_1\mu_2\ldots\mu_j}$ and $\mu_k = \begin{cases} \dfrac{N_S}{\mu_S}, if\ k \geq N_S \\ \dfrac{k}{\mu_S}, if\ 1 \leq k < N_S \end{cases}$.

A. Utilities of PN and SN in leasing periods

In a leasing period, since the SN can utilize the N_S leasing channels for secondary transmission, it may incur packet buffering cost and receive transmission benefits concurrently.

Denote by $\overline{N}_{S,L}$ the average number of SPs newly generated by the SN in a leasing period, by $S_{S,L}^{(k,y)}$, where $k \in [1, \overline{N}_{S,L}]$ and $y \in [0, n+k]$, the probability that the k^{th} SP newly generated in a leasing period sees exactly y SPs being buffered or in transmission, and by $p_{S,L}(x|y)$ the conditional probability that, when a SP newly generated in a leasing period sees exactly y SPs being buffered or in transmission, the next newly generated SP will see exactly x SPs. Obviously, $\overline{N}_{S,L} = \lceil\lambda_S E[T_L]\rceil$ and $p_{S,L}(x|y) = 0$ if $x > y+1$. From Figure 2, we have

$$S_{S,L}^{(k+1,x)} = \sum_{y=0}^{n+k} S_{S,L}^{(k,y)} p_{S,L}(x|y).$$

with $S_{S,L}^{(0,y)}$, $\forall\ y \geq 0$, being defined as $\pi_y(N_S, \frac{\lambda_S\mu_S}{N_S})$ in (1). According to Section 8.9.3 of [12], $p_{S,L}(x|y) = p_{y,x}$ in (5) with $Q = N_S$, $\mu = N_S/\mu_S$ and $\lambda = \lambda_S$.

Note that the SN has to buffer SPs when at least one of the k^{th}, $k \in [1, \overline{N}_{S,L} - 1]$, and $(k+1)^{st}$ SPs newly generated in a leasing period sees more than N_S SPs being buffered or in transmission. More specifically, given that the k^{th} newly generated SP sees y SPs and the $(k+1)^{st}$ one sees x SPs, if $y > N_S$ and $x \leq N_S$, then the SN should buffer totally $y+1-N_S$ SPs and the average buffering time for each of the $y+1-N_S$ SPs is $\frac{y+1-N_S}{2\lambda_S(y+1-x)}$; else, if $x > N_S$, since $y \geq x-1$, the SN should buffer an average number of $\frac{1}{2}(x+y+1-2N_S)$ SPs for a common average time length of $1/\lambda_S$. Thus the expected buffering cost of the SN during a leasing period can be expressed as:

$$E[B_{S,L}] = \sum_{k=1}^{\overline{N}_{S,L}-1}\Big\{\frac{c_S}{\lambda_S}\Big[\sum_{x=1}^{N_S}\sum_{y=N_S}^{n+k}\frac{(y+1-N_S)^2}{2(y+1-x)}S_{S,L}^{(k,y)}S_{S,L}^{(k+1,x)} + \sum_{x=N_S+1}^{n+k+1}\sum_{y=x-1}^{n+k}\frac{x+y+1-2N_S}{2}S_{S,L}^{(k,y)}S_{S,L}^{(k+1,x)}\Big]\Big\}, \quad (6)$$

where c_S denotes the cost for a SU to buffer one SP for a unit time.

Meanwhile, the expected transmission revenue of the SN during a leasing period can be expressed as:

$$E[R_{S,L}] = \sum_{k=1}^{\overline{N}_{S,L}-1}\Big[e_S\sum_{y=0}^{n+k}\sum_{x=0}^{y+1}(y+1-x)S_{S,L}^{(k,y)}S_{S,L}^{(k+1,x)}\Big], \quad (7)$$

where $y+1-x$ denotes the number of SPs that are transmitted between the generation of the k^{th} and $(k+1)^{st}$ SPs in the leasing period and e_S the average revenue of a SU in transmitting one SP.

On the other hand, since the PN in a leasing period can utilize the N_P reserved channels for primary communication, it also incurs buffering cost and receives transmission benefits concurrently. From (1), the average number of PPs buffered by the PN at the beginning of a leasing period is $\overline{I}_{P,L} = \sum_{k=0}^{\infty}k\pi_k(N, \frac{\lambda_P\mu_P}{N})$. Similar as (6), the average buffering cost of the PN in a buffering period can be expressed as:

$$E[B_{P,L}] = \sum_{k=1}^{\overline{N}_{P,L}-1}\Big\{\frac{c_P}{\lambda_P}\Big[\sum_{x=1}^{N_P}\sum_{y=N_P}^{\overline{I}_{P,L}+k}\frac{(y+1-N_P)^2}{2(y+1-x)}S_{P,L}^{(k,y)}S_{P,L}^{(k+1,x)} + \sum_{x=N_P+1}^{\overline{I}_{P,L}+k+1}\sum_{y=x-1}^{\overline{I}_{P,L}+k}\frac{x+y+1-2N_P}{2}S_{P,L}^{(k,y)}S_{P,L}^{(k+1,x)}\Big]\Big\}, \quad (8)$$

where $\overline{N}_{P,L}$ denotes the average number of PPs newly generated in a leasing period, $S_{P,L}^{(k,y)}$ for $k \in [1, \overline{N}_{P,L}]$ and $y \in [0, \overline{N}_{P,L}+k]$ the probability that the k^{th} PP newly generated in a leasing period sees exactly y PPs being buffered or in transmission, and c_P the cost for the PN to buffer one PP for a unit time. Obviously, $\overline{N}_{P,L} = \lceil\lambda_P E[T_L]\rceil$ and $S_{P,L}^{(k+1,x)} = \sum_{y=0}^{\overline{I}_{P,L}+k}S_{P,L}^{(k,y)}p_{P,L}(x|y)$, where $S_{P,L}^{(0,y)}$, $\forall y \geq 0$, is defined as $\pi_y(N_P, \frac{\lambda_P\mu_P}{N_P})$ in (1) and $p_{P,L}(x|y)$ denotes the conditional probability that, when a newly generated PP in a leasing period sees exactly y PPs, the next newly generated PP will see exactly x PPs. From [12], $p_{P,L}(x|y) = p_{y,x}$ in (5) with $Q = N_P$, $\mu = N_P/\mu_P$ and $\lambda = \lambda_P$.

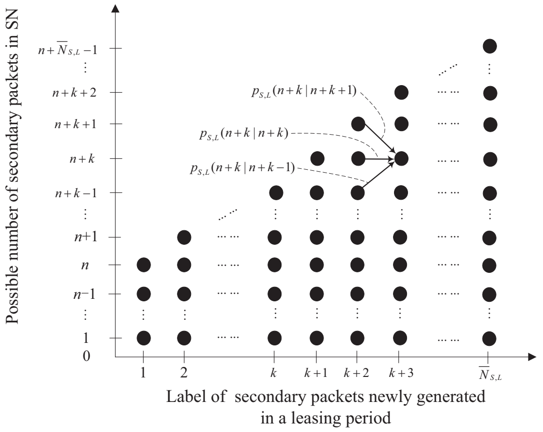

Fig. 2. Possible number of secondary packets (SPs) in the M/M/N_S queue seen by a SP newly generated in a leasing period.

Meanwhile, similar as (7), the expected transmission revenue of the PN during a leasing period can be expressed as:

$$E[R_{P,L}] = \sum_{k=1}^{\overline{N}_{P,L}-1} \left[e_P \sum_{y=0}^{\overline{I}_{P,L}+k} \sum_{x=0}^{y+1} (y+1-x) S_{P,L}^{(k,y)} S_{P,L}^{(k+1,x)} \right], \quad (9)$$

where e_P the average revenue of a PU in transmitting one PP.

B. Utilities of PN and SN in buffering periods

In a buffering period, the SN only accumulates packets at the rate λ_S and does not transmit any packet. Thus the average transmission revenue of the SN in this period is always 0 and the average buffering cost of the SN in a leasing period can be expressed as:

$$E[B_{S,B}] = \frac{c_S(1+2+...+n-1)}{\lambda_S} = \frac{c_S n(n-1)}{2\lambda_S}. \quad (10)$$

On the other hand, since the PN can utilize all N licensed channels for primary communication, it may incur buffering cost and receive transmission benefits concurrently. From (1), the average number of PPs buffered by the PN at the beginning of a buffering period is $\overline{I}_{P,B} = \sum_{k=0}^{\infty} k\pi_k(N, \frac{\lambda_P \mu_P}{N})$. Similar as (6), the buffering cost of the PN in a buffering period can be expressed as:

$$E[B_{P,B}] = \sum_{k=1}^{\overline{N}_{P,B}-1} \Bigg\{ \frac{c_P}{\lambda_P} \Bigg[\sum_{x=1}^{N} \sum_{y=N}^{\overline{I}_{P,B}+k} \frac{(y+1-N)^2}{2(y+1-x)} S_{P,B}^{(k,y)} S_{P,B}^{(k+1,x)} + \sum_{x=N+1}^{\overline{I}_{P,B}+k+1} \sum_{y=x-1}^{\overline{I}_{P,B}+k} \frac{x+y+1-2N}{2} S_{P,B}^{(k,y)} S_{P,B}^{(k+1,x)} \Bigg] \Bigg\} \quad (11)$$

where $\overline{N}_{P,B}$ denotes the average number of PPs newly generated by the PN in a buffering period and $S_{P,B}^{(k,y)}$ for $k \in [1, \overline{N}_{P,B}]$ and $y \in [0, \overline{N}_{P,B} + k]$ the probability that the k^{th} PP newly generated in a buffering period sees exactly y PPs being buffered or in transmission. Obviously, $\overline{N}_{P,B} = \lceil \lambda_P E[T_B] \rceil$ and $S_{P,B}^{(k+1,x)} = \sum_{y=0}^{\overline{I}_{P,B}+k} S_{P,B}^{(k,y)} p_{P,B}(x|y)$, where $S_{P,B}^{(0,y)}$, $\forall y \geq 0$, is defined as $\pi_y(N, \lambda_P \mu_P / N)$ in (1) and $p_{P,B}(x|y)$ denotes the conditional probability that, when a newly generated PP in a buffering period sees exactly y PPs, the next newly generated PP will see exactly x PPs. From [12], $p_{P,B}(x|y) = p_{y,x}$ in (5) with $Q = N$, $\mu = N/\mu_P$ and $\lambda = \lambda_P$.

Meanwhile, similar as (7), the expected transmission revenue of the PN during a buffering period is expressed as:

$$E[R_{P,B}] = \sum_{k=1}^{\overline{N}_{P,B}-1} \left[e_P \sum_{y=0}^{\overline{I}_{P,B}+k} \sum_{x=0}^{y+1} (y+1-x) S_{P,B}^{(k,y)} S_{P,B}^{(k+1,x)} \right]. \quad (12)$$

C. Average utilities of PN and SN

To stimulate the PN for joining traffic-adaptive spectrum leasing, the SN should pay a price to the PN for each leasing period. This payment consists of two parts, one being proportional to $E[T_L]$ and the other independent of it. More specifically, if the SN leases exactly N_S licensed channels from the PN in each leasing period at a leasing price p per channel per unit time, then the first payment for this leasing period is $pN_S E[T_L]$. On the other hand, to compensate for the PN's overhead to clear the N_S leasing channels before the beginning of each leasing period, the SN should pay a fixed charge K, $K > 0$, to the PN for each leasing channel, which is independent of $E[T_L]$. Thus the average payoff that the PN receives from the SN in each leasing period is $P_S = (K + pE[T_L])N_S$.

Based on this payoff as well as the the average transmission revenues and buffering costs of PN and SN in all buffering and leasing periods, the average utility of PN in traffic-adaptive spectrum leasing can be expressed as:

$$U_P(p, n, N_S) = \frac{P_S + E[R_{P,B}] + E[R_{P,L}] - E[B_{P,B}] - E[B_{P,L}]}{E[T_{pd}]}, \quad (13)$$

while the average utility of SN can be expressed as:

$$U_S(p, n, N_S) = \frac{E[R_{S,L}] - E[B_{S,B}] - E[B_{S,L}] - P_S}{E[T_{pd}]}. \quad (14)$$

IV. Equilibrium of traffic-adaptive spectrum leasing

To make the proposed traffic-adaptive spectrum leasing workable, the PN and SN should derive the equilibrium solution for the key parameters N_S, p and n in a distributed fashion. For this purpose, we formulate the proposed spectrum leasing scheme as a non-cooperative Stackelberg game [8],

$$p_{j,k} = \begin{cases} \int_0^\infty C_{j+1}^k (1-e^{-\mu t})^{j+1-k} (e^{-\mu t})^k \lambda e^{-\lambda t} dt, \ if \ Q \geq j+1 \geq k \\ \int_0^\infty e^{-Q\mu t} \frac{(Q\mu t)^{j+1-k}}{(j+1-k)!} \lambda e^{-\lambda t} dt, \ if \ j+1 \geq k \geq Q \\ \int_0^\infty \int_0^t C_Q^k (1-e^{-\mu(t-\tau)})^{Q-k} (e^{-\mu(t-\tau)})^k Q\mu e^{-Q\mu\tau} \frac{(Q\mu\tau)^{j-Q}}{(j-Q)!} \lambda e^{-\lambda t} d\tau dt, \ if \ k < Q < j+1 \\ 0, if \ k > j+1 \end{cases} \quad (5)$$

where the PN chooses N_S and p before the determination of n by the SN.

A typical way to determine the equilibrium in the Stackelberg spectrum leasing is by backward induction, which first calculates the best response $n^*(N_S, p)$ by the SN and then backtracks to the calculation of the optimal values N_S^* and p^* by the PN. However, as it is difficult to express n^* in terms of N_S and p by (14), we design the following algorithm for the PN to derive the possible equilibriums in the proposed traffic-adaptive spectrum leasing:

Algorithm 1. (The heuristic algorithm for deriving the equilibriums in traffic-adaptive spectrum leasing)
Step 1. Choose the minimal pricing interval $\triangle p$. Determine the minimal number $N_{S,\min}$ of leasing channels according to (2) and the maximal number $N_{S,\max}$ of leasing channels according to (3). Initialize $p = \triangle p$, $N_S = N_{S,\min}$, and $U_P^* = 0$.
Step 2. Initialize $n = 0$ and $U_S^* = 0$.
Step 3. Calculate the SN utility U_S according to (14). If $U_S > U_S^*$, then set $n^\sharp = n$ and $U_S^* = U_S$.
Step 4. If $U_S > 0$, then let $n = n + 1$ and return to Step 3.
Step 5. Set $n = n^\sharp$ and calculate the PN utility U_P according to (13). If $U_P > U_P^*$, then set $m = 1$; else, if $U_P = U_P^*$, then set $m = m + 1$; else, go to Step 7.
Step 6. Set $p^*(m) = p$, $N_S^*(m) = N_S$, $n^*(m) = n^\sharp$, and $U_P^* = U_P$.
Step 7. If $U_P > 0$, then let $p = p + \triangle p$ and return to Step 2.
Step 8. If $N_S < N_{S,\max}$, then let $N_S = N_S + 1$ as well as $p = \triangle p$ and return to Step 2.
Step 9. Output the m combinations $(p^*(1), N_S^*(1), n^*(1))$, $(p^*(2), N_S^*(2), n^*(2))$, ... , $(p^*(m), N_S^*(m), n^*(m))$ as the m equilibrium solutions for the traffic-adaptive spectrum leasing scheme. ∎

In summary, the PN exhausts all possible combinations of N_S and p for the corresponding best responses $n^*(N_S, p)$ of the SN so as to determine the optimal combination N_S and p for maximizing the PN utility. However, this Stackelberg spectrum leasing may have multiple equilibriums, i.e., there exist multiple choices of n that can achieve the maximum utilities of PN and SN. To remedy this, we further develop the following two rules for the PN and SN to derive a unique sub-game perfect equilibrium.

First, because the smaller the number of SPs the SN should buffer before a leasing period begins, the less the average transmission delay for SPs, and the better the QoS for secondary transmission, the SN is willing to apply the following rule for selecting a unique n:

(IV.A) For all possible values of n, which can maximize U_S in (14), the SN always prefers the smallest one.

Second, as the PN prefers less leasing channels to more leasing channels to better guarantee the QoS for primary transmission, it is willing to apply the following rule for selecting a unique N_S:

(IV.B) For all possible values of N_S, which can maximize U_P in (13), the PN prefers the smallest one.

Theorem 1. Following the rules (IV.A) and (IV.B), there exists a unique sub-game perfect equilibrium for Algorithm 1.

Proof. From (8), (9), (11), (12), and (13), the PN utility is always a linear increasing function of p and the SN utility a linear decreasing function of p. Thus, given a fixed value of n, the PN will always choose $p = p_{\max}$, where $p_{\max}$ is the maximal price that can make the SN utility $U_S > 0$. This, together with (IV.B), implies that, when n is fixed, there exists a unique combination (p, N_S) for the PN to maximize U_P. On the other hand, given a fixed combination of (p, N_S), the SN will always choose the smallest one among all values of n, which can maximize U_S in (14), by (IV.A). Thus (IV.A) and (IV.B) together give a unique (p, n, N_S), which is a sub-game perfect equilibrium for Stackelberg spectrum leasing. ∎

V. Numerical simulation

To evaluate the performance of the traffic-adaptive spectrum leasing (TASL) scheme proposed in Sections II~IV, we compare it with a so-called *partially traffic-adaptive spectrum leasing* scheme, or *PTASL* scheme in short, where the PN and SN negotiate a fixed time length for all leasing periods according to the statistic information of secondary traffics.

A. Description of the PTASL scheme

In the PTASL scheme, when the SN successfully leases N_S, $1 \le N_S \le N$, licensed channels from the PN by paying a unit price p, the infinite time of these channels will be synchronously divided into multiple frames. Similar as the TASL scheme illustrated in Figure 1, each frame is further divided into one buffering period for the SN to buffer SPs and one leasing period for the SN to transmit SPs over the N_S leasing channels. However, different from the TASL scheme of which the time length of each buffering or leasing period should adapt to the real-time secondary traffic, the PN and SN in the PTASL scheme only negotiate one fixed time length T_B for all buffering periods and the other fixed time length T_L for all leasing periods before the real spectrum leasing begins. In other words, all frames in the PTASL scheme share a common time length $T_{pd} = T_B + T_L$ and, before the beginning of spectrum leasing, the PN and SN should negotiate a ratio α, $0 \le \alpha \le 1$, for each leasing period in a frame, i.e., the time length of each leasing or buffering period in the PTASL scheme is fixed as $T_L = \alpha T_{pd}$ or $T_B = (1-\alpha)T_{pd}$, respectively.

Following the traffic parameters for SPs and PPs in Section II, the average number of SPs or PPs newly generated in a leasing period is $\overline{N}_{S,L} = \lceil \lambda_S \alpha T_{pd} \rceil$ or $\overline{N}_{P,L} = \lceil \lambda_P \alpha T_{pd} \rceil$, respectively, while the average number of SPs or PPs newly generated in a buffering period is $\overline{N}_{S,B} = \lceil \lambda_S (1-\alpha) T_{pd} \rceil$ or $\overline{N}_{P,B} = \lceil \lambda_P (1-\alpha) T_{pd} \rceil$. By substituting n in (6) and (7) as $\overline{N}_{S,B}$, we can first derive $E[B_{S,L}]$ in (6), $E[R_{S,L}]$ in (7), $E[B_{P,L}]$ in (8), $E[R_{P,L}]$ in (9), $E[B_{S,B}]$ in (10), $E[B_{P,B}]$ in (11), and $E[R_{P,B}]$ in (12) and then calculate the PN and SN utilities according to (13) and (14), respectively, where $E[T_L]$ is replaced by αT_{pd}. Moreover, since $\alpha = E[T_L]/E[T_{pd}]$, the constraints (2) and (3) for N_S can be transformed as:

$$max\{\frac{\overline{R}_{S,min}}{\overline{R}_{S,c} N_S}, \frac{\lambda_S \mu_S}{N_S}\} \le \alpha \le min\{\frac{N\overline{R}_{P,c} - \overline{R}_{P,min}}{\overline{R}_{P,c} N_S}, \frac{N - \lambda_P \mu_P}{N_S}\}. \quad (15)$$

In other words, when α is out of this range, the QoS requirement of either primary or secondary transmission cannot be satisfied.

To derive the equilibrium solution for N_S, p and α in a distributed fashion, we also formulate the PTASL scheme as a non-cooperative Stackelberg game, where the PN chooses N_S and p before the determination of α by the SN. Note that the unit price p can be interpreted as the cooperative relay service provided from the SN to the PN, e.g., the buffering period can be further divided into two sub-periods, one for the primary transmitters to send PPs to the SUs and the other for the SUs to relay the received PPs to the primary receivers. Thus the PTASL scheme in fact generalizes a class of the existing relay-based cooperative spectrum leasing schemes, e.g., [4] and [5].

B. Comparison between TASL and PTASL

Unless other specified, we set the utility parameters of PN and SN for both TASL and PTASL schemes as follows:

- $N = 8$, $N_S = 3$, $e_S = 4$, $e_P = 1$, $c_S = c_P = 1$, $K = 0.1$, $\lambda_P = 3packet/s$, $\lambda_S = 1.2packet/s$, $\mu_P = 10/7s$, $\mu_S = 10/9s$, $\overline{R}_{P,c} = 2800bit/s$, $\overline{R}_{S,c} = 4500bit/s$, $\overline{R}_{P,min} = 168000bit/s$, and $\overline{R}_{S,min} = 4500bit/s$

It can be verified that the constraints (2) and (3) are satisfied under this parameter setting. Thus the range of α in the PTASL scheme is $[0.44, 0.66]$ by (15). Let $\triangle p = 0.2$ for Algorithm 1 in Section IV. In the simulation of TASL and PTASL, we randomly generate the PPs or SPs, of which the transmission time follows the specified exponential distribution, according to the specified Poisson process for more than 100 alternations of leasing and buffering periods to yield average results.

Under the rules (IV.A) and (IV.B), Figure 3 depicts the average utilities of both PN and SN, respectively, in terms of the parameters p and n. More specifically, Figure 3(a) shows that, when $p = 2.2$ and $n = 3$, the utilities of both PN and SN in the TASL scheme reach their maximum values 4.53 and 1, respectively, while Figure 3(b) shows that, when $p = 1.2$ and $\alpha = 0.59$, the utilities of both PN and SN in the PTASL scheme reach their maximum values 3.98 and 0.87, respectively. Since the TASL scheme can afford the PN and SN with larger equilibrium average utilities than the PTASL scheme concurrently, both PN and SN are better off to adopt the TASL scheme instead of the PTASL scheme.

To illustrate the effect of p on the TASL and PTASL schemes, we fix T_{pd} in the PTASL scheme as the value of $E[T_{pd}]$ in the TASL scheme when $p = 0.4$. Figure 4 depicts both $E[T_{pd}]$ and $E[T_L]$ in terms of the price p charged by the PN. It shows that the SN in the PTASL scheme is willing to lease N_S licensed channels from the PN, i.e., $T_L \geq 0$, only when $p \in (0.4, 1.4)$, while the SN in the TASL scheme is always willing to join spectrum leasing, i.e., $E[T_L] \geq 0$, until $p \geq 2.4$. Thus, compared with the PTASL scheme, the TASL scheme enable the SN to lease N_S licensed channels at a wider range of leasing price. Under (IV.A) and (IV.B), Figure 5 depicts the average utilities of PN and SN in terms of the price p, where the unique sub-game perfect equilibrium of the TASL or PTASL scheme appears at $p = 2.2$ or $p = 1.2$, respectively. It shows that, if $p \in [0.4, 2.2]$, then the PN or SN in the TASL scheme can always achieve a larger average utility than the PN or SN in the PTASL scheme, respectively; else, if $p \geq 2.4$, then the PN and SN fail to reach a spectrum leasing agreement, i.e., the PN can only achieve an average utility by utilizing all N licensed channels for primary transmission and the SN utility reduces to zero. Thus, compared with the PTASL scheme, the TASL scheme can better motivate both PN or SN to join spectrum leasing.

To illustrate the effect of λ_S on the TASL and PTASL schemes, we fix T_{pd} in the PTASL scheme as the value of $E[T_{pd}]$ in the TASL scheme when $\lambda_S = 0.9$. Figure 6 depicts the average utilities of PN and SN at the sub-game perfect

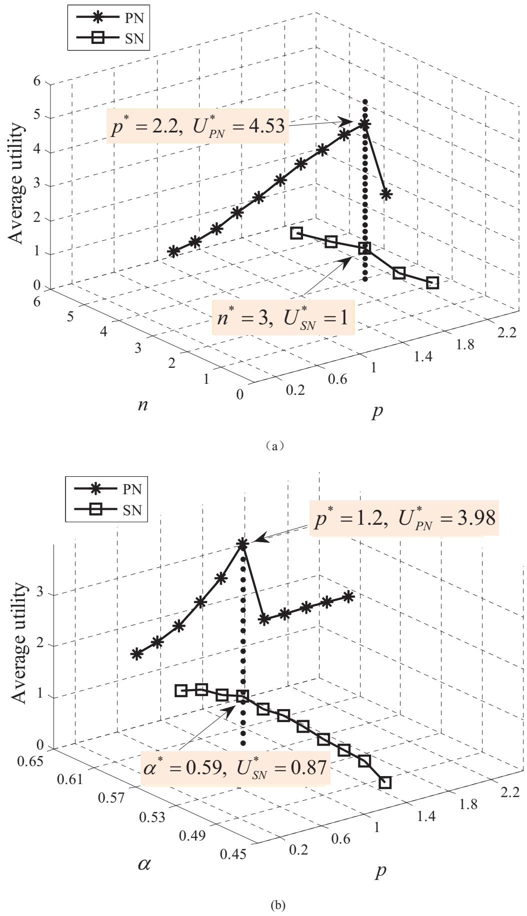

Fig. 3. (a) Average utilities of PN and SN vs the parameters p and n under the traffic-adaptive spectrum leasing (TASL) scheme; (b) average utilities of PN and SN vs the parameters p and α under the partially traffic-adaptive spectrum leasing (PTASL) scheme.

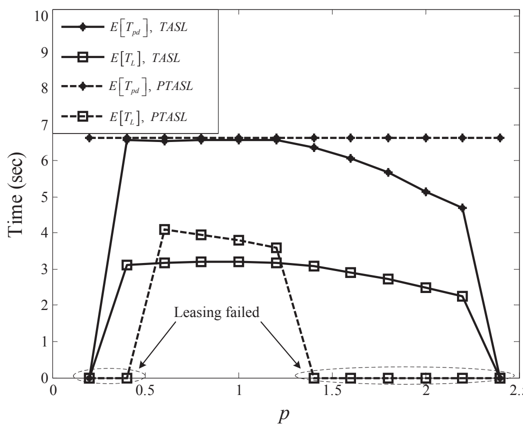

Fig. 4. $E[T_{pd}]$ and $E[T_L]$ vs the leasing price p.

equilibrium in terms of λ_S. It shows that the PN in the TASL scheme can always achieve a larger average utility than that in the PTASL scheme, while the SN in the TASL scheme can achieve a smaller average utility than that in the PTASL scheme when $\lambda_S \leq 1.1$ and a larger utility than the latter

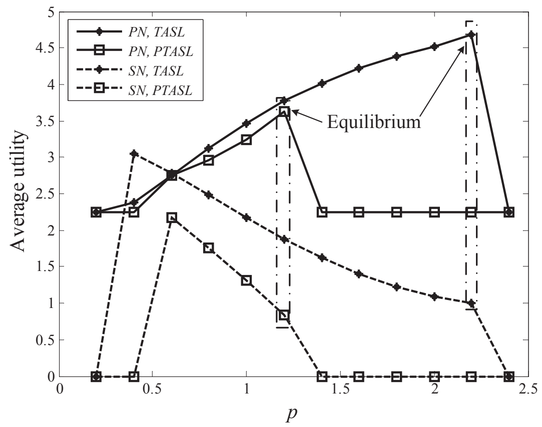

Fig. 5. Average utilities of PN and SN vs the leasing price p.

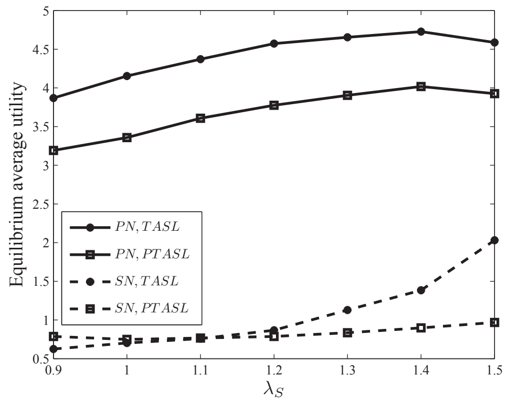

Fig. 6. Equilibrium average utilities of PN and SN vs the arrival rate λ_S of secondary packets.

otherwise. Thus the heavier the secondary traffic load, the easier for the TASL scheme to form a leasing agreement between PN and SN. Figure 7 depicts the utilization ratio of N licensed channels in terms of λ_S. It shows that, when $\lambda_S \in [0.9, 1.5]$, the utilization ratio of the TASL scheme is always larger than that of the PTASL scheme. Thus the TASL scheme enables the PN and SN to utilize limited channel resources in a more efficient fashion than the PTASL scheme.

VI. Conclusion

The present paper proposes a novel spectrum leasing scheme between one primary network (PN) and one secondary network (PN) by fully adapting the time length of spectrum leasing to the real-time secondary traffics, formulates the average utilities of PN and SN in the traffic-adaptive spectrum leasing (TASL), and derives an unique sub-game perfect equilibrium solution for this scheme based on the Stackelberg game mode under two rules that can enhance the QoS for primary and secondary traffics. Numerical simulation shows that, compared with the partially traffic-adaptive spectrum leasing scheme, which generalizes the existing spectrum leasing schemes based on cooperative relay, e.g., [4] and [5], the proposed TASL scheme can concurrently improve the transmission utilities of both PN and SN, encourage the PN and SN to reach a mutually beneficial spectrum leasing agreement

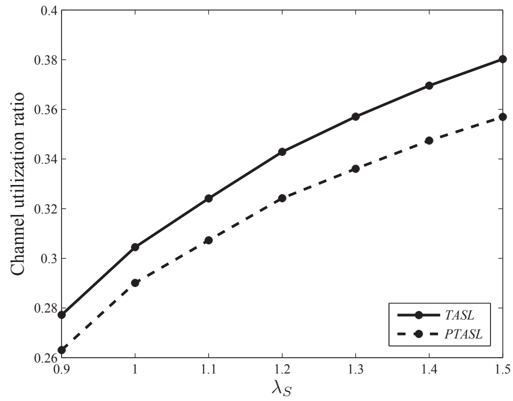

Fig. 7. Utilization ratio of N licensed channels vs the arrival rate λ_S of secondary packets.

over a wider range of leasing price and for a heavier secondary traffic load, and enable them to utilize the limited spectrum resources in a more efficient fashion. While the proposed TASL scheme is only applicable to the SN under centralized control, the theoretical framework for the PN and SN utilities formulated in this paper could serve as the basis for future traffic-adaptive spectrum leasing under distributed control.

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